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THE EQUILIBRIUM AND STABILITY OF THE GASEOUS COMPONENT OF THE GALAXY. IV.

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ABSTRACT

The stability of a self-gravitating, non-rotating, plane-parallel, isothermal gas layer with equipartition magnetic and cosmic-ray components, immersed in a rigid isothermal layer of stars, is considered with respect to waves with motions perpendicular to the $\vec{B}_e - \vec{g}_e$ plane, where $\vec{B}_e$ and $\vec{g}_e$ are the equilibrium magnetic and gravitational field vectors. The magnetic field and cosmic-ray gas hinder gravitational instability, increasing the minimum length necessary to produce instability by the factor $(1 + \alpha + \beta)^{1/2}$, where α is the ratio of magnetic pressure to gas pressure and β is the ratio of cosmic-ray pressure to gas pressure.

I. INTRODUCTION

In Paper I of this series (Kellman 1972a) we calculated the distribution above the galactic plane of an isothermal, plane-parallel layer of gas with equipartition magnetic and cosmic-ray components, immersed in an isothermal layer of stars. In Paper II (Kellman 1972b) we considered the gravitational stability of the gas layer with respect to plane and axially-symmetric perturbations, neglecting however the effects of the magnetic and cosmic-ray components. The result was that the presence of a stellar component increased the radius of the marginally unstable state in the symmetry plane, but only slightly. In Paper III (Kellman 1972c) the stability analysis was modified by the inclusion of a one-dimensional equipartition magnetic field. When disturbances propagate across the magnetic field, the minimum length necessary to produce gravitational instability is increased by the factor $(1 + \alpha)^{1/2}$, where α is the ratio of magnetic pressure to gas pressure. No such simple expression is obtained when disturbances propagate along the magnetic field.

It is our purpose here to include an equipartition cosmic-ray gas in the analysis. We consider only the stability of waves (with wave vector $\vec{k}$) propagating across the magnetic field. Motions are constrained to be perpendicular to the $\vec{B}_e$ - $\vec{g}_e$ plane, where $\vec{B}_e$ and $\vec{g}_e$ are the equilibrium magnetic and gravitational field vectors. As we stressed in Papers II and III, the stability analysis may shed light (i) on the existence of large-scale structure (1 - 2 kpc, 10^7 $M_\odot$) observed in the gaseous component of spiral arms in the Galaxy (McGee and Milton 1964), and (ii) on the excitation of density waves in the Galaxy by Jeans' type instabilities in the gas layer beyond the corotation distance (Lin 1970), ultimately leading to the formation of spiral structure.

II. STABILITY ANALYSIS

The relation between the vectors $\vec{B}_e$, $\vec{g}_e$, and $\vec{k}$ and the xyz coordinate system is shown in Figure 1. The basic equations to consider are the continuity, momentum, hydromagnetic, Poisson, and heat equation, plus an additional equation expressing the fact that the cosmic-ray pressure is a constant of the motion, which follows from the fact that the sound speed of the cosmic-ray gas is much greater than any other wave velocity considered here (Parker 1966; Field 1970):

$$\frac{d}{dt}\rho_g + \rho_g \nabla\cdot\vec{v} = 0 \qquad (1)$$

$$\nabla p_g + \rho_g \frac{d}{dt}\vec{v} - \frac{1}{4\pi}\vec{B}\cdot\nabla\vec{B} + \frac{1}{8\pi}\nabla B^2 + \rho_g \nabla\phi = 0 \qquad (2)$$

$$\frac{\partial}{\partial t}\vec{B} - \nabla\times(\vec{v}\times\vec{B}) = 0 \qquad (3)$$

$$4\pi G(\rho_g + \rho_*) - \nabla^2(\phi_g + \phi_*) = 0 \qquad (4)$$

$$p_g = \langle v_{tz}^2\rangle \rho_g \qquad (5)$$

$$\frac{d}{dt} p_{c-r} = 0. \tag{6}$$

ρ, p, ϕ, and $\vec{B}$ are, respectively, the density, pressure, gravitational potential, and magnetic field strength. $\vec{v}$ is the gas velocity, a first order quantity. $\langle v_{tz}^2 \rangle$ is the mean square z turbulent gas velocity dispersion. The subscripts g, *, and c-r denote gas, star, and cosmic-ray, respectively. Perturbations of the form

$$\rho_g = \rho_{eg} + \Delta\rho_g \tag{7}$$

$$p_g = p_{eg} + \Delta p_g = \langle v_{tz}^2 \rangle \rho_g \tag{8}$$

$$p_{c-r} = p_{ec-r} + \Delta p_{c-r} \tag{9}$$

$$\phi_g = \phi_{eg} + \Delta\phi_g \tag{10}$$

$$\phi = \phi_e + \Delta\phi_g = \phi_{eg} + \phi_{e*} + \Delta\phi_g \tag{11}$$

$$\vec{B} = \vec{B}_e + \Delta\vec{B} \tag{12}$$

are applied to equations (1)-(6), and terms only to first order in the perturbed quantities are retained, with the result that

$$\frac{\partial}{\partial t} \Delta\rho_g + \vec{v}\cdot\nabla\rho_{eg} + \rho_{eg} \nabla\cdot\vec{v} = 0 \tag{13}$$

$$\nabla\phi_e \Delta\rho_g + \langle v_{tz}^2 \rangle \nabla\Delta\rho_g + \rho_{eg} \frac{\partial}{\partial t} \vec{v} + \nabla\Delta p_{c-r} + \frac{1}{4\pi} \nabla (\vec{B}_e \cdot \Delta\vec{B}) + \rho_{eg} \nabla\Delta\phi_g = 0 \tag{14}$$

$$\frac{\partial}{\partial t}\Delta\vec{B} + \vec{v}\cdot\nabla\vec{B}_e + \vec{B}_e\,(\nabla\cdot\vec{v}) - (\vec{B}_e\cdot\nabla)\,\vec{v} = 0 \tag{15}$$

$$\frac{\partial}{\partial t}\Delta p_{c\text{-}r} + \vec{v}\cdot\nabla p_{ec\text{-}r} = 0 \tag{16}$$

$$4\pi G\Delta\rho_g - \nabla^2\Delta\phi_g = 0. \tag{17}$$

The subscript 'e' refers to the equilibrium quantities; Δ refers to the perturbed quantities.

The coefficients of equations (13)-(17) are all independent of t, x, and y, enabling us to Fourier analyze in these variables ($\partial/\partial t \to n$, $\partial/\partial x \to ik_x = 0$, $\partial/\partial y \to ik_y$), with the result that, written in component form, equations (13)-(17) become

$$n\Delta\rho_g + ik_y\rho_{eg}v_y + \left(\rho_{eg}\frac{\partial}{\partial z} + \frac{d}{dz}\rho_{eg}\right) v_z = 0 \tag{18}$$

$$ik_y\,\langle v_{tz}^2\rangle\,\Delta\rho_g + n\rho_{eg}v_y + ik_y\,\Delta p_{c\text{-}r} + ik_y\frac{B_e}{4\pi}\Delta B_x + ik_y\,\rho_{eg}\,\Delta\phi_g = 0 \tag{19}$$

$$\langle v_{tz}^2\rangle\,\frac{\partial}{\partial z}\Delta\rho_g + \Delta\rho_g\frac{d}{dz}\phi_e + n\rho_{eg}v_z + \frac{\partial}{\partial z}\Delta p_{c\text{-}r} + \frac{B_e}{4\pi}\,\frac{\partial}{\partial z}\Delta B_x$$

$$+\frac{1}{4\pi}\Delta B_x\,\frac{d}{dz}B_e + \rho_{eg}\,\frac{\partial}{\partial z}\Delta\phi_g = 0 \tag{20}$$

$$ik_y\,B_e\,v_y + \left(B_e\frac{\partial}{\partial z} + \frac{d}{dz}B_e\right) v_z + n\Delta B_x = 0 \tag{21}$$

$$v_z \frac{d}{dz} p_{ec-r} + n\Delta p_{c-r} = 0 \qquad (22)$$

$$-4\pi G\Delta\rho_g + \left(\frac{\partial^2}{\partial z^2} - k_y^2\right) \Delta\phi_g = 0. \qquad (23)$$

Equations (19) and (20) are the y and z components of the momentum equation; equation (21) is the x component of the hydromagnetic equation.

As in Paper III, we assume that

$$\frac{d}{dz} \rho_{eg} = f\rho_{eg}, \qquad (24)$$

from which it follows that

$$\frac{d}{dz} B_e = \frac{1}{2} fB_e \qquad (25)$$

and

$$\frac{d}{dz} p_{ec-r} = fp_{ec-r}, \qquad (26)$$

since B_e^2/ρ_{eg} and p_{ec-r}/ρ_{eg} are assumed to be independent of z. Equation (24) is consistent with the presence of a stellar component. Further, one can show that

$$\frac{d}{dz} \phi_e = - \langle v_{tz}^2 \rangle \ f \ (1 + \alpha + \beta) \qquad (27)$$

$$\frac{1}{B_e} \frac{\partial}{\partial z} \Delta B_x = \left(\frac{\partial}{\partial z} + f/2\right) \frac{\Delta B_x}{B_e} \qquad (28)$$

and

$$\frac{1}{\rho_{eg}} \frac{\partial}{\partial z} \Delta\rho_g = \left(\frac{\partial}{\partial z} + f\right) \frac{\Delta\rho_g}{\rho_{eg}}, \tag{29}$$

where $\alpha = B_e^2/8\pi \rho_{eg} \langle v_{tz}^2 \rangle$ is the ratio of magnetic pressure to gas pressure and $\beta = p_{ec-r}/\rho_{eg} \langle v_{tz}^2 \rangle$ is the ratio of cosmic-ray pressure to gas pressure. We restrict the analysis to the marginally unstable state by setting $n = 0$. In addition, we define the variables ϵ, δ, γ, and ψ by the equations

$$\epsilon = \Delta\rho_g/\rho_{eg} \tag{30}$$

$$\delta = \Delta B_x/B_e \tag{31}$$

$$\gamma = \Delta p_{c-r}/\langle v_{tz}^2 \rangle \tag{32}$$

$$\psi = \Delta\phi_g/\langle v_{tz}^2 \rangle. \tag{33}$$

With these various substitutions and restrictions, the system (18)-(23) reduces to

$$\epsilon + \gamma/\rho_{eg} + 2\alpha\delta + \psi = 0 \tag{34}$$

$$\left(\frac{\partial}{\partial z} - f\alpha - f\beta\right) \epsilon + \frac{1}{\rho_{eg}} \frac{\partial}{\partial z} \gamma + 2\alpha \left(\frac{\partial}{\partial z} + f\right) \delta + \frac{\partial}{\partial z} \psi = 0 \tag{35}$$

$$\frac{-4\pi G \rho_{eg}}{\langle v_{tz}^2 \rangle} \epsilon + \left(\frac{\partial^2}{\partial z^2} - k_y^2\right) \psi = 0. \tag{36}$$

One can easily show that

$$\frac{1}{\rho_{eg}} \frac{\partial \gamma}{\partial z} = \left(\frac{\partial}{\partial z} + f\right) \frac{\gamma}{\rho_{eg}}, \tag{37}$$

and defining the variable τ by the equation

$$\tau = \gamma/\rho_{eg} + 2\alpha\,\delta, \tag{38}$$

the system (34)-(36) becomes

$$\epsilon + \tau + \psi = 0 \tag{39}$$

$$\left(\frac{\partial}{\partial z} - f\alpha - f\beta\right)\epsilon + \left(\frac{\partial}{\partial z} + f\right)\tau + \frac{\partial}{\partial z}\psi = 0 \tag{40}$$

$$\frac{-4\pi G\rho_{eg}}{\langle v_{tz}^2\rangle}\epsilon + \left(\frac{\partial^2}{\partial z^2} - k_y^2\right)\psi = 0. \tag{41}$$

We differentiate equation (39) and subtract equation (40), with the result that

$$\tau = (\alpha + \beta)\,\epsilon, \tag{42}$$

and from equation (39) it follows that

$$\epsilon = -\frac{\psi}{(1 + \alpha + \beta)}. \tag{43}$$

Equations (41) and (43) are combined to yield a single second order differential equation in the variable ψ:

$$\frac{\partial^2}{\partial z^2}\psi + \left[\frac{1}{2H_g^2\,(1 + \alpha + \beta)}\,\frac{\rho_{eg}}{\rho_{ego}} - k_y^2\right]\psi = 0, \tag{44}$$

where ρ_{ego} is the value of ρ_{eg} at the symmetry plane z = 0. H_g is a scale height for the gas distribution in the z direction in the limit that $\alpha \to 0$ and $\beta \to 0$, and has been defined by equation (13) of Paper II:

$$H_g^2 = \frac{\langle v_{tz}^2 \rangle}{8\pi G \rho_{ego}}. \tag{45}$$

The appropriate boundary conditions to impose on equation (44) are

$$\frac{\partial}{\partial z}\psi\,(z = 0) = 0 \tag{46}$$

$$\lim_{|z| \to \infty} \psi = 0. \tag{47}$$

Equation (46) results because $\psi = \Delta\phi_g \,/\, \langle v_{tz}^2 \rangle$ is an even function of z; equation (47) results because $\Delta\phi_g$ is constrained to $\to 0$ as $|z| \to \infty$.

Equations (44), (46), and (47) define an eigenvalue problem in the sense that only certain discrete values of k_y will result in ψ's that satisfy (a) equation (44) and (b) the boundary conditions imposed by equations (46) and (47). We have discussed the nature of this eigenvalue problem in some detail in Paper II. Here we need only note that in the limit that $\alpha \to 0$ and $\beta \to 0$, equation (44) reduces to equation (18) of Paper II. Therefore, to compute the radius r_1 of the marginally unstable state (proportional to $1/k_y$), we simply choose an appropriate value for $\langle v_{tz}^2 \rangle^{1/2}$ and read off the value of r_1 from Figure 1 of Paper II, calculated with

ρ_{ego} = 0.025 $M_{\odot}/pc^3$, ρ_{*o} = 0.064 $M_{\odot}/pc^3$, and $\langle v_{*z}^2 \rangle^{1/2}$ = 18 km/sec. The modification introduced by the magnetic and cosmic-ray components is obtained by multiplying this r_1 by the factor $(1 + \alpha + \beta)^{1/2}$. The principal result then to emerge from this study is that the presence of equipartition magnetic and cosmic-ray components tends to stabilize the gas layer against gravitational instability, increasing the radius of the marginally unstable state in the symmetry plane by the factor $(1 + \alpha + \beta)^{1/2}$. In Table 1 we compare the Ledoux radius (Ledoux 1951) (ρ_{*o} = 0, B_{eo} = 0, p_{ec-ro} = 0), r_1 calculated in Paper II (ρ_{*o} = 0.064 $M_{\odot}/pc^3$, B_{eo} = 0, p_{ec-ro} = 0), r_1 calculated in Paper III (ρ_{*o} = 0.064 $M_{\odot}/pc^3$, B_{eo} = 3 μG, p_{ec-ro} = 0), and r_1 calculated here (ρ_{*o} = 0.064 $M_{\odot}/pc^3$, B_{eo} = 3 μG, p_{ec-ro} = 0.50 × 10^{-12} dynes/cm^2), each computed as a function of $\langle v_{tz}^2 \rangle^{1/2}$.

III. DISCUSSION

Choosing $\langle v_{tz}^2 \rangle^{1/2}$ = 7.5 km/sec and ρ_{ego} = 1 H atom/cm^3 = 0.025 $M_{\odot}/pc^3$, values typically quoted in the literature, the Jeans' radius λ_J for an infinite uniform gas is 0.638 kpc. All subsequent modifications and improvements to the Jeans' analysis tend to increase λ_J. Specifically, a self-consistent z distribution according to the formula $\rho_{eg}(z)/\rho_{ego}$ = sech^2 (z/H) increases λ_J to λ_L = 0.902 kpc (Ledoux 1951), an increase of 41%. The inclusion of a rigid stellar layer with ρ_{*o} = 0.064 $M_{\odot}/pc^3$ and $\langle v_{*z}^2 \rangle$ = 18 km/sec increases λ_L, though only slightly, to r_1 = 0.953 kpc (Kellman 1972b), an increase of 6%. The inclusion of an equipartition magnetic field increases r_1 by the factor

$(1 + \alpha)^{1/2}$, which for $B_{eo} = 3\,\mu G$ gives 1.10 kpc (Kellman 1972c), an increase of 15%. Finally, a combined equipartition magnetic field and cosmic-ray gas increases r_1 by the factor $(1 + \alpha + \beta)^{1/2}$, which for $B_{eo} = 3\,\mu G$ and $p_{ec-ro} = 0.50 \times 10^{-12}$ dynes/cm^2 gives 1.31 kpc, an increase of 37%.

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TABLE 1

RADIUS OF THE MARGINALLY UNSTABLE STATE AS A FUNCTION OF

$\langle v_{tz}^2 \rangle^{1/2}$, ρ_{*o}, B_{eo}, AND $p_{ec\text{-}ro}$

$\langle v_{tz}^2 \rangle^{1/2}$ (km/sec)	r_1 (Ledoux) (kpc) $\begin{pmatrix} \rho_{*o} = 0 \\ B_{eo} = 0 \\ p_{ec\text{-}ro} = 0 \end{pmatrix}$	r_1 (kpc) $\begin{pmatrix} \rho_{*o} = 0.064 \frac{M_\odot}{pc^3} \\ B_{eo} = 0 \\ p_{ec\text{-}ro} = 0 \end{pmatrix}$	r_1 (kpc) $\begin{pmatrix} \rho_{*o} = 0.064\ M_\odot/pc^3 \\ B_{eo} = 3\,\mu G \\ p_{ec\text{-}ro} = 0 \end{pmatrix}$	r_1 (kpc) $\begin{pmatrix} \rho_{*o} = 0.064\ M_\odot/pc^3 \\ B_{eo} = 3\,\mu G \\ p_{ec\text{-}ro} = 0.5 \times 10^{-12}\ \text{dynes/cm}^2 \end{pmatrix}$
2.5	0.301	0.318	0.67	0.96
5.0	0.601	0.635	0.87	1.10
7.5	0.902	0.953	1.10	1.31
10.0	1.202	1.270	1.40	1.56
15.0	1.803	1.905	2.03	2.11
20.0	2.404	2.540	2.61	2.69

FIGURE CAPTIONS

1. The relation between $\vec{B}_e$, $\vec{g}_e$, and $\vec{k}$ and the xyz coordinate system.

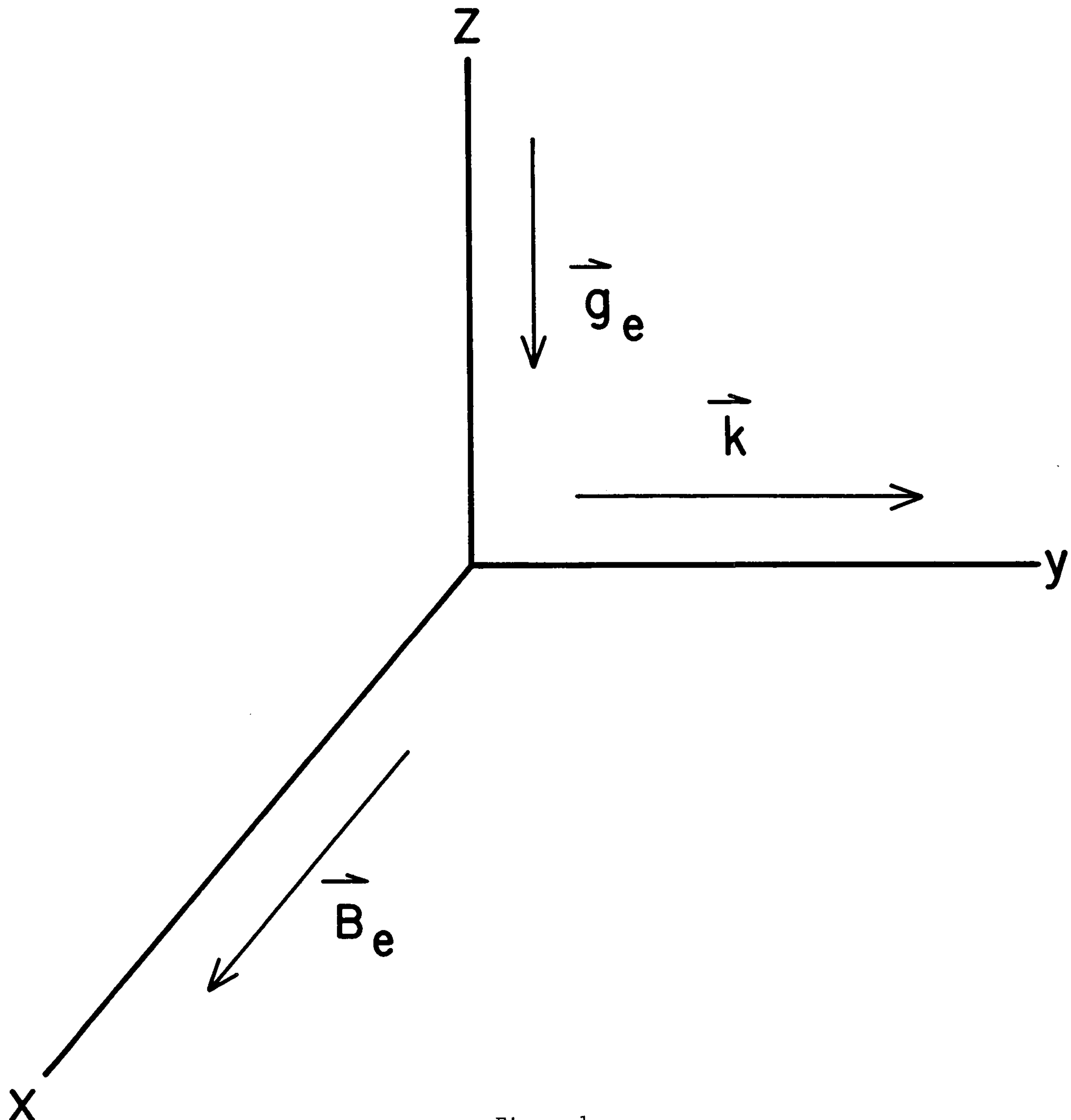

Figure 1